ent thickness for the stressed and unstressed regions, Eq. (A13) reduces to Eq. (6).

For the case of field fringing the capacitance of each region differs from the one-dimensional capacitance.

Let

$$\lambda = C_\lambda / C,$$

where C_λ is the capacitance between the wavefront and the electrodes including field fringing effects, C is the capacitance for one-dimensional field, and λ is the fringing factor.

In the manner of the one-dimensional derivation, we derive the current, including field fringing effects. The current is

$$i_\lambda = PA\{\lambda_1\lambda_2[C_2(dC_1/dt) - C_1(dC_2/dt)] + C_2C_1(\lambda_2 d\lambda_1/dt - \lambda_1 d\lambda_2/dt)\}(\lambda_2C_2+\lambda_1C_1)^{-2}. \quad \text{(A14)}$$

In order to numerically evaluate the effect of fringing on the current, measurements of the capacitance of various diameter-to-thickness disks were made. From these data we determined approximate values for λ and $d\lambda/dt$. Our calculations showed that the effect of this type of field fringing was to depress the current jump below the value for the one-dimensional case with the extent of the effect depending upon the d/l ratio for the disk. This result is in agreement with the experimental observations on the full electrode gauges.

Electromechanical Coupling

The elastic stiffness of the quartz changes slightly with time due to electromechanical coupling. We evaluate this effect below.

The piezoelectric equation of state for one-dimensional strain along the x axis is

$$\sigma = c_{11}^E s - e_{11}E. \quad \text{(A15)}$$

Substituting the field from Eq. (A5) into Eq. (A15), we find that

$$\sigma = c_{11}^E s + e_{11}P(1-U_s t/l)/\epsilon = s[c_{11}^E + e_{11}^2(1-U_s t/l)/\epsilon]. \quad \text{(A16)}$$

From the one-dimensional wave equation we know that

$$\rho U_s^2 = \partial\sigma/\partial s; \quad \text{(A17)}$$

hence

$$\rho U_s^2 = c_{11}' = c_{11}^E[1+e_{11}^2(1-U_s t/l)/c_{11}^E\epsilon]. \quad \text{(A18)}$$

When $t=0$ we find that

$$c_{11}' = c_{11}^E[1+e_{11}^2/c_{11}^E\epsilon] = c_{11}^D,$$

and when $t=l/U_s$

$$c_{11}' = c_{11}^E = c.$$

Bechmann finds $c_{11}^E = 86.74\times10^{10}$ dyn-cm^{-2} and $c_{11}^D = 87.49\times10^{10}$ dyn-cm^{-2}. This analysis demonstrates that the elastic stiffness is slightly time dependent due to electromechanical coupling. A more extensive analysis by Baerwald[18] indicates that the electromechanical coupling will cause the current to increase by a factor $e^2/c\epsilon$ during wave transit time.

---

[18] H. G. Baerwald, Sandia Laboratory (private communication).